The Possibility of Social Choice

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Abstract

In 1951 Kenneth Arrow published a book in which he proved that social choice was impossible. There was no way to amalgamate individual preferences into a social preference in such a way that certain rational and normative conditions were met. Later Gibbard and Satterthwaite proved that any such amalgamation of individual preferences in which there was no advantage to any individual to use strategy to order their preferences insincerely in order to get a better result for themselves was impossible or led to the selection of a dictator. These impossibility theorems have been thought to rule out direct democracy and also welfare economics giving credibility to the implication that representative democracy and capitalist economics are the best systems that can be devised.

Instead of simple amalgamation, we have devised a more general information processing system which represents the implementation of a mechanism that accepts inputs from individual choosers as utilitarian ratings and outputs a social choice in the form of a complete ordinal ranking. It is a hybrid utilitarian approval system. This system is designed to disincentivize choosers from choosing strategically or insincerely. The system itself maximizes the efficacy of each individual input. It is utility based, but processes the information in such a way as to alleviate concerns about interpersonal comparisons of utility. It provides a rationale as to where to draw the line between approved and unapproved candidates. It also satisfies Arrow's five rational and normative conditions while making the mechanism even more robust normatively. The result is that a utility based social choice system has been devised which negates both impossibility theorems and should give new life to welfare economics and direct democracy as well as making a contribution to the literature on approval voting.

Introduction

In Social Choice and Individual Values, Kenneth Arrow (1951: p. 1) wrote "In a capitalist democracy there are essentially two methods by which social choices can be made: voting, typically used to make 'political' decisions, and the market mechanism, typically used to make 'economic' decisions." Initially, Arrow does not distinguish between political and economic systems claiming that both are means of formulating social decisions based on individual inputs. Arrow then purports to show that there is no rational way to make social decisions based on the amalgamation of individual ones, assuming certain rational and normative conditions are met, thus ruling out welfare economics, economic democracy and direct political democracy. The dichotomy between political and economic systems remains with the implication that representative democracy and capitalist economics are the best systems that can be devised. Arrow's result, formerly called the *paradox of voting,* was first discovered by the Marquis de Condorcet (1785). Condorcet's paradox shows that majority preferences can become intransitive when there are three or more alternatives. Arrow basically mathematized Condorcet's insight.

Jackson (2001: p. 2) states: "Often, one thinks of the desired outcomes as the given and analyzes whether there exist game forms for which the strategic properties induce individuals to (always) choose actions that lead to the desired outcomes." We design a game form for which the strategic properties induce individuals to choose actions that lead to the desired outcome – a possible social choice – while disincentivizing them from choosing strategically as individuals. We show that this mechanism also satisfies Arrow's rational and normative conditions. Gibbard and Satterthwaite concurred with Arrow and proved that any social choice system that was strategy proof was also impossible.
Gibbard (1973: p. 587) states: "An individual 'manipulates' the voting scheme if, by misrepresenting his preferences, he secures an outcome he prefers to the 'honest' outcome - the choice the community would make if he expressed his true preferences." Satterthwaite showed that the requirement for voting

procedures of strategyproofness and Arrow's requirements for social welfare functions are equivalent: a one-to-one correspondence exists between every strategy-proof voting procedure and every social welfare function satisfying Arrow's five requirements. The mechanism presented in this paper represents a strategy-proof voting procedure assuming that individuals will seek to strategize in such a way as to increase expected social utility for themselves. This mechanism also satisfies Arrow's rational conditions while making his normative conditions even more robust.

Gibbard's results were based only on the possibility that someone could use strategy if they were astute enough to stumble on a way to do so. (1973: p. 590) "Note that to call a voting scheme manipulable is not to say that, given the actual circumstances, someone is really in a position to manipulate it." Only the possibility exists in an elaborate mathematical structure. Gibbard doesn't assume that there is any formularizable or identifiable strategy that a voter could use to manipulate the system. Other writers have pointed out this difficulty: (Meir et. al.: p. 149) "In other words, computational complexity may be an obstacle that prevents strategic behavior." By contrast, we analyze a situation in which an actual identifiable strategy exists which can be known both to the individual chooser and to the system, which amalgamates or processes the choices, itself. If the system does the strategizing for each individual, there is no incentive for an individual to do so.

Gibbard's and Satterthwaite's analysis is deterministic while the problem of manipulability is inherently probabilistic. In an actual election it would be impossible for a voter to know the ideal strategy unless they knew how every other voter was going to vote. Polling, however, can provide some information of a probabilistic nature about other voters. We incorporate the fundamentally probabilistic nature of the choosing process in our analysis, and the mechanism we develop is generalizable to the situation in which polling data is available.

The following simple example presages the path forward. Let's say there are two alternatives and a number of individual choosers. Each individual chooser specifies their input as utilities on a scale which is the real line between "0" and "+1". Then the utilities are summed over all choosers, and the alternative with the highest sum is determined to be the winner. Furthermore, let's say individual 1 has a utility of 0.8 for alternative A and 0.2 for alternative B. The strategy involved would lead individual 1 to change their sincere utility rating for alternative A to "+1' and, similarly, candidate B to "0". This would maximize the chances that A would win based just on individual 1's choice alone and would tend to maximize individual 1's expected utility in the social outcome. However, if the information processing system, which accepts inputs from the choosers, does the strategy for them and processes the choice as "+1" for A and "0" for B based on individual 1's sincere utility ratings, then there is no incentive for this individual to misrepresent their input, and they can go ahead and submit their sincere utility ratings as 0.8 for alternative A and 0.2 for alternative B. This indicates the path forward when more than 2 alternatives are under consideration. Of course, the chooser could misrepresent their utilities giving A "+1" and B, "0", but there would be nothing gained from doing so since the system does it for them. For more complex systems, individual choosers might actually tend to diminish their satisfaction with the outcome if they represented their choices insincerely. It is assumed that the system does this calculation for every individual chooser, not just individual 1.

Aki Lehtinen (2011: p.376) concludes that Arrow's Impossibility Theorem is not relevant in the final analysis: “Arrow’s impossibility result and the closely related theorems given by Gibbard and Satterthwaite are unassailable as deductive proofs. However, we should not be concerned about these results because their most crucial conditions are not justifiable. Fortunately, we know that strategy-proofness is usually violated under all voting rules and that IIA [Independence of Irrelevant Alternatives] does not preclude strategic voting.” Unlike Lehtinen we do not dispute the Arrow and Gibbard-Satterthaite analyses and conclusions in this paper. Their mathematics is impeccable. Instead, by thinking outside the box, we analyze a social choice mechanism which accomplishes what Arrow, Gibbard and Satterthwaite purportedly set out to accomplish – a system that produces a social choice based on individual inputs which exemplifies certain rational and normative criteria including strategyproofness. The mechanism analyzed here accomplishes this in a manner that not only is more realistically implementable in terms of actual voting/choosing systems but is also more robust normatively.

A major stumbling block for the development of utilitarian social choice systems regards the issue of interpersonal comparisons. It has been thought that scales which measure the utilities of individuals are incompatible, and that any scale chosen, upon which all individuals are supposed to rate their utilities, would be arbitrary. Arrow (1951: p. 9) states: "The viewpoint will be taken here that interpersonal comparison of utilities has no meaning and, in fact, that there is no meaning relevant to welfare comparisons in the measurability of individual utility." Thus, according to Arrow, any individual input must be based on individual preference rankings of the form aRbRc, meaning a is preferred or indifferent to b, b is preferred or indifferent to c etc. Although "comparisons in the measurability of individual utilities" may have no meaning when done by an outside observer, the assertion of utilities by individuals themselves on a scale of their own choosing certainly does.

We assume that choosers can place their respective utilities for alternatives on a scale of their own choosing within the set of all real numbers, $\mathbb{R}$, and also choose the end points. In general there will be a utility for each possible alternative specified by each chooser. We will show that, for the information processing mechanism modeled here, any affine linear transformation of an individual's set of utility ratings will yield the same output or social choice results, and, therefore, it doesn't matter which scale an individual chooses. This is not to say that the utility scale chosen by an individual is not meaningful to the individuals themselves, but just that, whatever scale they choose, their contribution to the final output of the system we analyze will be the same.

We develop a social choice mechanism that is utility based, but which overcomes the objections of arbitrariness of utility scales, is strategyproof and also meets an upgraded version of Arrow's normative and rational criteria. Therefore, social choice is not impossible, and the possibility of other such systems or mechanisms exists.

Utilitarian and Approval Choosing

Utilitarian and approval choosing are exactly analogous to utilitarian voting (UV) and approval voting (AV), and, therefore, “voting” and “choosing” are used interchangeably for the purposes of this paper. Also the words “alternative” and “candidate” will be used interchangeably.

Arrow sets up the problem so that each individual chooser orders or ranks all alternatives and then society is required to come up with an ordering that is best according to his stated criteria. He states (Arrow, 1951: p. 11-12) “In the theory of consumer's choice each alternative would be a commodity bundle; ... in welfare economics, each alternative would be a distribution of commodities and labor requirements. … in the theory of elections, the alternatives are candidates.”

The method constructed in this paper inputs or uploads information from the individual choosers in the form of preference ratings or utilities and outputs information in the form of complete social preference rankings from which social ratings can be derived since the underlying individual ratings are known. From these social preference rankings, the mechanism we analyze produces an unordered winning set, W of size m, consisting of those alternatives with the top m rankings. Furthermore, we can compute the utility of the winning set for each voter since we know from their input how they rated each member. Summing utilities over all voters would give the social utility of the winning set.

In order to negate the Gibbard-Sattertwaite theorems, which maintain that every choosing system for which an individual chooser can use strategy to improve the outcome for themselves violates Arrow's conditions, we choose a social choice processing system or mechanism which itself implements the optimum strategy for each individual assuming that that strategy consists of voting in such a way as to maximize the expected average utility of the winning set for themselves. The system we describe here involves placing an individualized threshold in the monotonically increasing and unrestricted utility set associated with the candidates which is submitted by each individual chooser. Each candidate above this threshold is given an approval style vote of "+1", and each candidate below threshold is given an approval style vote of "0". As the threshold increases, there are less candidates above threshold and the average utility rating of the set of candidates above threshold increases. Conversely, as the threshold decreases, the number of candidates above threshold increases while the average utility of the set of candidates above threshold decreases. We choose the optimum threshold to be just under that utility such that the average utility of the set of candidates above threshold is maximum.

Claude Hillinger (2005: pp. 295-321) has made the case for utilitarian voting: “There is, however, another branch of collective choice theory, namely utilitarian collective choice, that, instead of fiddling with Arrow’s axioms, challenges the very framework within which those axioms are expressed. Arrow’s framework is *ordinal* in the sense that it assumes that only the information provided by individual orderings over the alternatives are relevant for the determination of a social ordering. Utilitarian collective choice assumes that individual preferences are given as *cardinal* numbers; social preference is defined as the sum of these numbers.” The difference between Hillinger's statement and the mechanism considered here is that social preference is *not* defined as the sum of cardinal numbers. There is a unique transformation done by the information processing system or mechanism itself *for each voter* from their cardinal inputs to their contribution to the approval style output. Hence, the system we examine is a utilitarian approval hybrid.

Lehtinen (2015: p.35) has shown that "strategic behavior increases the frequency with which the *utilitarian winner* is chosen compared to sincere behavior ". Therefore, the mechanism described in this paper should accomplish two things: sincere voting behavior on the part of individuals *and* increased selection of the utilitarian winner or winners compared to other voting systems. While Lehtinen abandons the Arrow and Gibbard-Sattherwaite conditions in the interests of increased social utility, strategyproofness is not violated if the system itself applies the strategy instead of the individual choosers. Lehtinen (2015: p. 39) also argues that interpersonal comparisons "can be made in a methodologically acceptable way in evaluating the performance of voting rules if the same comparison is made under every voting rule." The system presented in this paper manifests the fact that the same comparison is made for every voter, and, therefore, it should be "methodologically acceptable" to use Lehtinen's term. The issue of interpersonal comparisons is demonstrably moot for the implementation of the social choice mechanism considered here.

Formal Statement of System Parameters

We first define the following sets:

i) $V = \{v_1, v_2, \dots v_q\}$ is a set of voter/choosers, where $v_j \in V$ denotes the j^{th} chooser.

ii) $C = \{c_1, c_2, \dots, c_n\}$ is the ordered set of candidates; candidates appear on the ballot in $c_1, c_2, \dots, c_n$ order. $c_i \in C$ denotes the i^{th} candidate.

iii) $X = \{x_1, x_2, \dots x_n\}$ $x_i = \{\mathbb{N}^0\}$, the set of non-negative integers. X represents the cumulative votes for candidates as they appear on the ballot.

iv) $Y = \{y_1, y_2, \dots y_n\}$ is the set which orders the candidates by the number of votes received by each candidate. $y_1 R y_2 R \dots R y_n$. R means "is preferred or indifferent to."

v) $W = \{w_1, w_2, \dots, w_m\}$ is an unordered set of candidates of size m < n representing the winning set.

vi) $C_j = \{c_{1j}, c_{2j}, \dots c_{nj}\}$ is the set of preferences of alternatives for the j^{th} voter.

vii) $B_j = \{b_{1j}, b_{2j}, \dots, ,b_{nj}\}$ is a set of approval style votes in order of the j^{th} voter's candidate preferences. $b_{ij} = \{ \mathbb{N}^0 \mid 0, 1\}$

viii) $U_j = \{u_{1j}, u_{2j}, \ldots u_{nj}\}$ is a set of utilities of size n, with $u_{1j} \geq u_{2j} \geq \ldots \geq u_{nj}$ and $0 \leq u_{ij} \leq 1$, $\forall$ i,j. U_j is the utility set of the jth voter. This assumes an affine linear transformation from $u_{ij} \in \{\mathbb{R} \mid -\infty < u_{ij} < +\infty\}$ as will be explained later.

ix) $T_j = \{t_{1j}, t_{2j}, \ldots t_{nj}\}$ is a set of thresholds of size n such that $t_{1j} \geq t_{2j} \geq \ldots \geq t_{nj}$ and $0 \leq t_{ij} \leq +1$, $\forall\, i, j$.

x) $U_{aj} = \{u_{a1j}, u_{a2j}, \ldots, u_{anj}\}$ is the set of utilities above threshold for each chooser. u_{aij} is defined as the sum of utilities above threshold t_{ij} for voter j, $\forall$ i,j . The sum of utilities above threshold is computed for each of the n thresholds. n_{aij} is the corresponding number of utilities above threshold.

We now define following functions:

i) $\tau: C \rightarrow X$ defines an ordered pair, (c_i, x_i) such that $\tau(c_i) = x_i$

ii) $\alpha: X \rightarrow Y$ α defines an ordered pair (x_r, y_r) such that [if $x_r \geq x_z$ then $y_r \mathrm{R} y_z$] for $1 \leq r, z \leq n$, *r , z integers*

iii) β: $Y \rightarrow W$ such that $\beta(\mathrm{y_i}) = w_i$ for $1 \leq i \leq m$

iv) $\pi: W \rightarrow C$ defines an ordered pair (w_i, c_i) such that $\pi(w_i) = c_i$ for $1 \leq \mathrm{i} \leq m$

v) $\chi_j: C \rightarrow C_j$ The function χ_j assigns to each element $c_i \in C$ an element $\chi_j(c_i) = c_{ij}$ such that $c_{1j} \mathrm{R} c_{2j} \ldots \mathrm{R} c_{nj}$ for $1 \leq \mathrm{j} \leq \mathrm{q}$ where R means "is preferred or indifferent to". Each voter, j, orders the set of alternatives according to their preferences.

vi) η_j: $C_j \rightarrow U_j$ the function η_j assigns to each element $c_{ij} \in C_j$ an element $\eta_j(c_{ij}) = u_{ij}$ where u_{ij} is the utility that is assigned to candidate c_{ij} by voter j.

vii) $\delta_j: C_j \rightarrow B_j$ defines an ordered pair (c_{ij}, b_{ij}) such that $\delta_j(c_{ij}) = b_{ij}$ for $1 \leq j \leq q$

viii) γ_j: $U_j \rightarrow T_j$ defines the relationship $\gamma_j(u_{ij}) = t_{ij}$ *such that* $t_{ij} = u_{ij} - \varepsilon$ *where* $\varepsilon \ll 1, \forall\ i, j$

ix) $\phi_{aj}: T_j \rightarrow U_{aj}$ such that $\phi_{aj}(t_{ij}) = u_{aij}$, where $u_{aij} = \sum_{u_{ij} > t_{ij}} u_{ij} \quad \forall\ i, j$ n_{aij} is the

corresponding number of utilities above threshold.

The probability, p, of k above threshold candidates being in the winning set due to chance alone is

$$\mathbf{p} = \frac{\binom{\mathbf{n}_{aij}}{\mathbf{k}}\binom{\mathbf{n} - \mathbf{n}_{aij}}{\mathbf{m} - \mathbf{k}}}{\binom{\mathbf{n}}{\mathbf{m}}}$$

This is the hypergeometric function (Wikipedia) which is the mathematics for a ball and urn problem in which the urn contains n_{aij} white balls associated with candidates with utilities above threshold and $n - n_{aij}$ black balls associated with candidates with utilities below threshold. **p** equals the probability of k white balls drawn from the urn out of m total draws, without replacement, from a finite population of size n, wherein each draw can either produce a white ball or a black ball. m balls are drawn and placed in the winning set, W. m is the size of the winning set. n is the total number of candidates with associated utilities for each individual voter. We assume no prior knowledge or polling information regarding candidate probabilities although the analysis can be generalized to the case where polling information is available. Exactly which white ball (associated with a particular candidate) is picked is not known so that we use the average utility of above threshold candidates, u_{aij}/n_{aij}, in subsequent calculations.

Let u_{Wj} be a random variable which represents the average utility of above threshold candidates in the winning set for voter j so that $0 \leq u_{Wj} \leq 1$, $\forall\ i, j$. Then the expected value of average utility of above threshold candidates in the winning set for voter j at threshold t_{ij} is given by

$$E_{t_{ij}}\left(u_{w_j}\right) = \sum_{k=1}^{s}\left[\frac{{}^{n_{aij}}C_k \times {}^{n_{bij}}C_{m-k}}{{}^{n}C_m}\right]\left[\frac{u_{aij}}{n_{aij}}\right]$$

where $s = min\{m, n_{a\,i\,j}\}$

Let t_j^* be the optimal threshold which is the threshold which results in the maximization of the expected value of average utility $E_{t_{ij}}(u_{W_j})$ of the winning set, W, for each individual j. n_j^* is the corresponding number of candidates with utilities above that threshold. So

$$E_{t_j^*}\left(u_{w_j}\right) = max\left\{E_{t_{ij}}\left(u_{w_j}\right)\right\}$$

As the threshold is decreased from $t_j^\star$, the average utility of the winning set for voter j decreases because there are more above threshold utilities with lower values under consideration. However, the probability of an above threshold candidate being in the winning set increases. As the threshold is increased from $t_j^\star$, the probability of an above threshold candidate being in the winning set decreases. However, the average utility of the set of candidates above threshold increases.

Candidates whose utilities are greater than the optimal threshold, $t_j^\star$, will be given the maximum vote of "+1", and candidates whose utilities are less than $t_j^\star$ will be given the minimum vote of "0" $\forall\, j$.

Strategy

The strategy, **σ,** counts the votes for each candidate:

σ:

for z = 1, n
$x_z = 0$
end z (initializes X)
for j = 1,q
for i = 1, n
$b_{ij} = 0$ (initializes B_j)
if $\{u_{ij} > t_j^*$ then
$b_{ij} = 1$

$$x_i = x_i + \tau\chi_j^{-1}\delta_j^{-1}(b_{ij})$$

end i

end j

end **σ**

Let ${}^{A}u_j$ be the utility of the winning set, *W*, for voter/chooser j post-election, and ${}^{A}u$ be the social utility of the winning set for all voter/choosers - the utility of the social choice.

$$ {}^{A}u_j = \sum_{i=1}^{m} \eta_j \chi_j \pi_j (w_i) $$

$$ {}^{A}u = \sum_{j=1}^{q} {}^{A}u_j $$

Optimal Threshold Social Choice

The Optimal Threshold Social Choice (OTSC) Information Processing System is an implementation of a mechanism which can be modeled as follows:

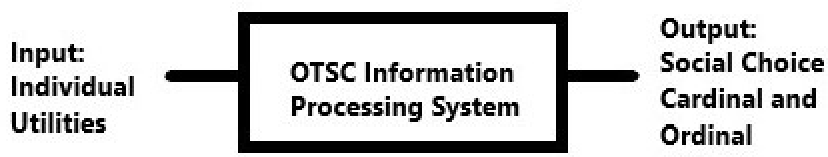

Figure 1

The OTSC system uses the above analysis to optimize each individual's choice so that they are disincentivized from choosing insincerely. It overcomes Gibbard-Satterthwaite's concerns about strategic choosing by individuals while meeting Arrow's rational and normative conditions as proven below. It even upgrades Arrow's normative conditions since more finely tuned cardinal input information is used while Arrow's analysis only involved less precise ordinal information. The key is that individuals are disincentivized from voting insincerely because the OTSC system strategizes for them. The optimal strategy maximizes the expected value of average utility of the winning set, W, for each voter/chooser based on their vote/choice alone. The assumption of utility maximizing is made by other writers (Lehtinen, 2008: pp. 688-704): "Under strategic behaviour voters are assumed to maximise expected utility ... ". The voter's input is the ordered set of candidates C_j and the associated ordered set of utilities U_j. The output is the set Y consisting of the ordered set of all candidates by vote totals from which is derived the winning set, W, which is unordered and consists of $m < n$ candidates. It is assumed that each individual voter specifies an unrestricted, utilitarian style input which represents their sincere utility ratings for candidates in set, C.

Examples

We have computed expected average utility for utility profiles U1 and U2 (dropping the j). We have plotted $E_{t_{ij}}(u_{W_j})$ (simplifying notation to E(u)) vs threshold T) for n =21, 1 ≤ i ≤ 21, m = 1 - 4 as shown in Figures 2 and 3. Figure 2 represents a "smooth transition" between utilities. Figure 3 represents an "abrupt transition" between utilities.

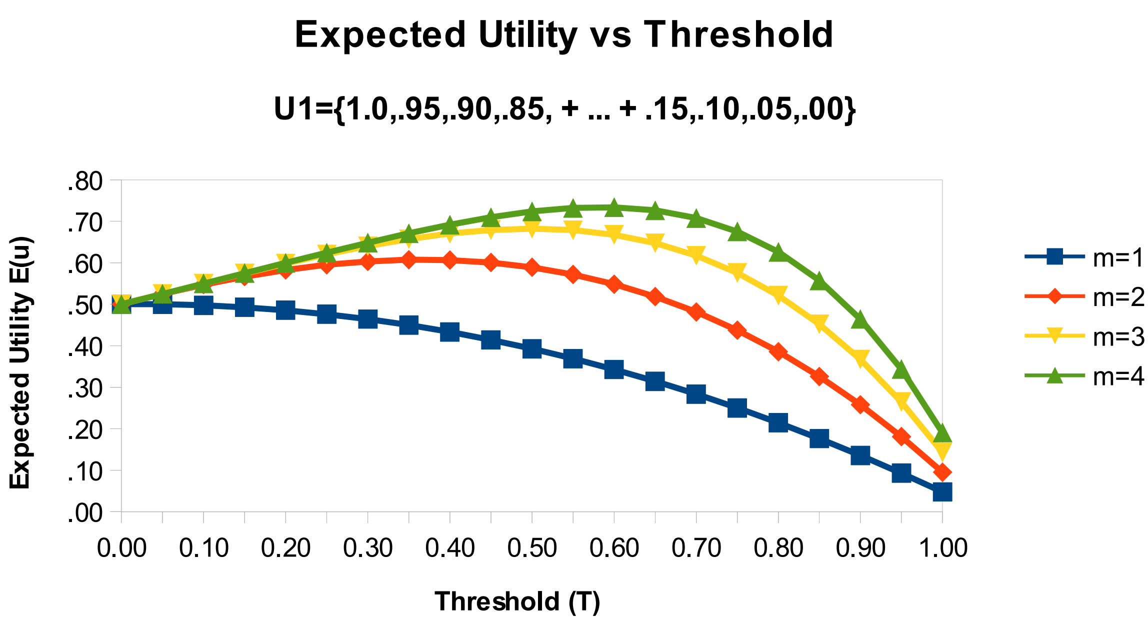

Figure 2

For m=1, E(u) max = .5000 @ T = 0.00. For m=2, E(u) max = .6075 @ T = 0.35.
For m=3, E(u) max = .6823 @ T = 0.50. For m=4, E(u) max =.7338 @ T = 0.60

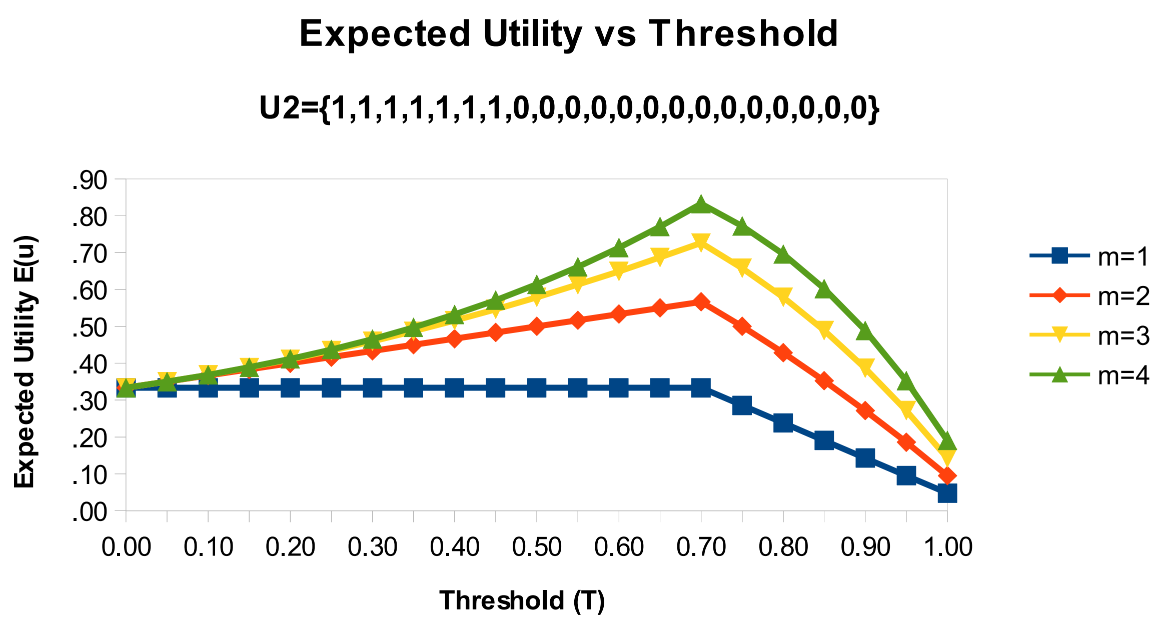

Figure 3

For m=1, E(u) max = 0.3333 @ T = 0.7. For m=2, E(u) max = 0.5667 @ T = 0.7.
For m=3, E(u) max = 0.7263 @ T = 0.7. For m=4, E(u) max = 0.8327 @ T = 0.7.

We define t_j^* as the greatest value of T such that *E(u)* is a maximum i.e. limsup[*E(u)*] for $0 < T < 1$, as shown in Figure 3 for *m* = 1. Figure 3 shows that the threshold for this utility profile is always at 0.7 regardless of the value of m which is intuitively plausible. As m increases, the expected utility approaches +1.

Figure 2 shows that for utility profile U1 and m = 1 the best strategy is to give an approval style vote of "1" to all candidates except the one whose utility is "0". That one gets an approval style vote of "0". As the size of the winning set increases, however, fewer candidates are assigned an approval style vote of "+1", and the expected utility of the winning set for the voter with this utility profile increases.

Smith (2005) proves the following: “Mean-based thresholding is optimal range-voting strategy in the limit of a large number of other voters, each random independent full-range.” Range voting is similar to utilitarian voting. While Smith's analysis assumes a completely randomized set of utility profiles, it does not give the optimal strategy for any particular utility profile. Lehtinen (2010: pp. 285-310) has also used expected utility maximizing voting behavior to indicate which candidates should be given an approval style vote. He agrees with Smith that an approval style vote of "+1" should be given to all candidates for whom their utility exceeds the average utility of all candidates and a "0" otherwise. Both Smith and Lehtinen consider only single member districts.

Based on the examples in Figues 2 and 3 we would disagree with Lehtinen.With regard to Figure 2, the average utility is 0.5, but our results show a maximum expected utility at a threshold of 0.0 for m = 1, and progressively higher optimal thresholds for higher values of m. For Figure 3 the optimal threshold is 0.7 for all values of m with maximum expected utiity increasing as m increases. However, the average utility is 7/21 = 0.33. If the threshold for "+1" approval votes were to be set to 0.33 as Lehtinen suggests, the expected utility would be decreased significantly from what it is at the optimal threshold of 0.7.

The OTSC Mechanism Satisfies Arrow's Five Conditions

Arrow's five rational and normative conditions are

1) Unrestricted domain.

2) Positive Association of Individual and Social Values

3) Independence of Irrelevant Alternatives (IIA)

4) Citizens' Sovereignty

5) Non-dictatorship

In general since any alternative, c_{ij}, can be given any utility rating, $u_{ij} \in \mathbb{R} \;\; \forall$ i,j. by each individual voter/chooser, number (1) is satisfied. An affine linear transformation so that $0 \leq u_{ij} \leq 1$, which is the assumed input to OTSC, will not change the outcome. The results will be the same no matter which utility scale each individual chooses since the optimal threshold is a function of n_j*. Any affine linear transformation of a chooser's utility scale will yield the same results since n_j* will be the same before and after the transformation. Let an individual express their utilities on a scale of their choice on the real line.

Number (2) is satisfied because raising some alternative's utility, u_{ij}, in an individual's utilitarian style input from just under to just above optimal threshold will result in that alternative's receiving one more approval style choice, b_{ij}, in the final summation, *X*. This would raise the social choice result by one for that alternative potentially putting that alternative in the winning set and/or changing the ordering in the set, *Y*. Similarly, lowering a candidate's rating in some individual's utility scale might eliminate that alternative from the winning set or change the ordering of the set, *Y*. Number (4) is satisfied since the OTSC system treats all alternatives and citizens in an equal and neutral manner. Number (5) is satisfied since the winning set is based only on individual inputs in such a way that no individual has any more say over the outcome than any other individual.

As for number (3), IIA, first of all utilitarian style sincere ratings for each candidate are assumed to be independent of each other regardless of the composition of the alternative set. (Hillinger, 2004: p. 3), "A cardinal number assigned to an object indicates its place on a scale that is independent of other objects." So if an individual rates a candidate at a particular rating on their utility scale, and then another candidate enters or leaves the race, it is assumed that the first candidate will still be rated the same. A candidate's dropping out or entering the race is assumed not to change an individual's sincere ratings for the other candidates.

Now consider the case in which, after the election occurs, a candidate dies or drops out.
Arrow (1951: p. 26) states : “Suppose that an election is held, with a certain number of candidates in the field, each individual filing his list of preferences, and then one of the candidates dies. Surely the

social choice should be made by taking each of the individual's preference lists, blotting out completely the dead candidate's name, and considering only the orderings of the remaining candidates in going through the procedure of determining a winner.” Arrow implies that the voting has already occurred, but the final determination of the winner(s) has not been made. If this were the case, the OTSC system would blot out the dead candidate's rating from all of the individual rating scales, recompute all the individual thresholds and recompute the ordered outcome, *Y*, and the winning set, *W*. Therefore, the dead candidate is not irrelevant, just not included in the final computation.

Now consider the case in which a new candidate enters the race after the balloting has occurred but before the election results have been published. The added utility rating for that candidate would be submitted/uploaded to the OTSC system by each individual chooser after the utilities for the other candidates had presumably already been submitted, and the results had already been computed. The OTSC system would then recompute the individual thresholds including the added candidate's utility rating and the final social choice results would then be recomputed. The individual choosers would not have an incentive to rate the added candidate insincerely knowing that the OTSC system would give them the strategically best outcome based on the complete list of submitted utilities. Therefore, candidate add-ons would not incentivize any individual chooser to choose insincerely. Furthermore, compliance with IIA is satisfied for add-ons since ratings for two candidates at a time could be uploaded for each individual chooser with thresholds recomputed at each step or as a final step thus demonstrating that the social choice can be arrived at by pairwise comparisons which Arrow's IIA demands.

Optimal Threshold Social Choice is Strategyproof

Since the data is processed in an optimal manner for each individual chooser by the system itself, giving each chooser the optimal strategy, the choosers have no incentive to misrepresent their preferences or to choose insincerely. They would either choose sincerely or the OTSC system might process their input in such a way as to give them a suboptimal result. There is no advantage to individuals to misrepresent their preference ratings. The choosers are disincentivized from choosing insincerely. The strategy has been placed in the processing of the choices rather than in each individual chooser's hands.

The optimum strategy for each individual is to vote in such a way as to maximize their expected average utility for the winning set. This is done by the OTSC system itself by setting an optimal threshold in each individual's utility style input so that each candidate above threshold receives the maximum "vote" and every candidate below threshold receives the minimum "vote". This maximizes the expected value of average utility of the social choice for each individual based on that individual's choice alone. This effectively turns the utilitarian style inputs into approval style outputs, but the connection with the underlying utilitarian basis of the system is maintained since the original utilities are known and can be used to compute the utility of the social choice for each individual and for society as a whole.

The Issue of Interpersonal Comparisons is Moot

Arrow (1951: p. 10) dwells on the fact that individual utility scales are not compatible. He compares them with the measurement of temperature which is based on arbitrary units and the arbitrary terminal points of freezing and boiling for the Celsius scale and completely different end points for the Fahrenheit scale. "Even if, for some reason, we should admit the measurability of utility for an individual, there still remains the question of aggregating the individual utilities. At best, it is contended that, for an individual, their utility function is uniquely determined up to a linear transformation; we must still choose one out of the infinite family of indicators to represent the individual, and the values of the aggregate (say a sum) are dependent on how the choice is made for each individual. In general, there seems to be no method intrinsic to utility measurement which will make the choice compatible."

As Arrow suggests, we take into account that each individual has a unique utility function. There is no need to (Arrow: p. 12) “choose one out of the infinite family of indicators to represent the individual.” Each individual gets to choose their own indicator. Let's say that, in general, utility can be measured as points u_{ij} on $\mathbb{R}$. It's up to the individual chooser where to place the points, including the end points, corresponding to the utilities of each candidate in the candidate set consisting of n candidates $C = \{c_1, c_2, \ldots, c_n\}$. Let's call the end points of some individual's utility scale u_{max} and u_{min}. This will define the scale. There needs not be an actual utility assigned to either of these end points. Since the OTSC system optimizes the utility of the social choice for each individual, there would be an optimal threshold above which all utilities are changed to the maximum value and below which all utilities are converted to the minimum value.

For the OTSC system in particular, the results will be the same no matter which utility scale each individual chooses since the optimal threshold is a function of n^*_{aij}. Any linear transformation of a chooser's utility scale will yield the same results since n^*_{aij} will be the same before and after the transformation. Let an individual express their utilities on a scale of their choice on the real line. For the sake of the analysis we do an affine linear transformation to convert each individual's input utility scale to one with end points "0" and "1". $f(u) = au + b$ with a,b integers represents an affine linear transformation of the individual's input scale. Let $f(u_{max}) = +1 = au_{max} + b$ and $f(u_{min}) = -1 = au_{min} + b$. It follows that $a = 2/(u_{max} - u_{min})$ and $b = -(u_{max} + u_{min})/(u_{max} - u_{min})$.

Consequently, Arrow's statement that “the values of the aggregate are dependent on how the choice is made for each individual” is not true. The choice is not *made* for each individual; each individual makes their *own* choice. However, since any scale chosen by each individual will yield the same results, without loss of generality, we can standardize the choosing process by transforming individual scales to the real line between "0" and "+1" before input to the OTSC system.

Amartya Sen (2002: p. 71) stated "... economists came to be persuaded by arguments presented by Lionel Robbins and others (deeply influenced by "logical positivist" philosophy) that interpersonal comparisons of utility had no scientific basis. 'Every mind is inscrutable to every other mind and no common denominator of feelings is possible.' Thus, the epistemic foundations of utilitarian welfare economics were seen as incurably defective." The OTSC system demonstrates that there is a sound epistemic basis for a utility based social choice mechanism. Therefore, it is in fact logical positivist *because* it has a sound scientific basis. Showing that Arrow's and Gibbard-Satterthwaite's impossibility results are invalid for just one mechanism such as OTSC proves that social choice is not impossible potentially for other mechanisms as well.

"The difficult we do right now, the impossible will take a little while" (from "Crazy He Calls Me" by Carl Sigman and Bob Russell.)

Preference Rankings Can Be Converted to Ratings and Vice Versa

Arrow's assumption of input preference orderings or rankings for each individual is a tacit assumption of equal utility scales for each individual equivalent to the "one man, one vote" principle. With the assumption that individual orderings represent equally spaced utilities, we can convert orderings or rankings to ratings. This may or may not be a very accurate representation of the underlying utilities, but it's the best information available if only individual orderings are known. These ratings can then be used as inputs to the OTSC mechanism.

The available information for rankings is of the form aRbRcRd... . For the system considered here and without loss of generality, any scale with any end points can be used for this conversion procedure as long as the preference ratings are equally spaced. For instance, we can choose the real line between "0" and "+1". We let the top ranked candidate be placed at "+1" and the lowest ranked candidate be placed at "0". The other candidates then would be equally spaced on the scale. The OTSC information processing system will then output approval style positive choices for those candidates represented by utilities above the optimal threshold and zero choices for those candidates represented by utilities below the optimal threshold for each individual. As we have shown, any affine linear transformation of an individual's utility scale will not change the results of the OTSC mechanism. The outputs are in the form of integers and represent the votes or choices for each alternative or candidate. Thus individual inputs can be in the form of rankings if utility information is not available. Therefore, the OTSC social choice inputs and outputs can both be be represented as rankings (orderings) and/or ratings (utilities).

Conclusions

It has been shown that social choice is possible thus disproving and replacing both Arrow's and Gibbard-Satterthwaite's impossibility theorems which are in essence mathematical tautologies devoid of the inherently probabilistic nature of voting methods and which do not assume that individual choices can be processed in any other way than by simple addition. Their results apply to certain deterministic mathematical structures and were not extended to the probabilistic case considered here. Rather than disproving these impossibility theorems mathematically, we have developed a completely new concept, the Optimal Threshold Social Choice (OTSC) mechanism, based on implementation theory which accepts Arrow's and Gibbard-Satterthwaite's conditions and yet produces actual possible results. Furthermore, we assume an upgraded and more robust version of Arrow's normative conditions. The OTSC system accepts individual utilitarian style preference ratings as inputs and outputs approval style social choice preference rankings. It processes the inputs in such a way as to maximize the expected utility of the social choice for each individual chooser based on their choices alone. This is done by setting an optimal threshold in the input utilitarian data of each individual chooser and outputting "+1" approval style choices for those candidates above threshold and "0" approval style choices for those candidates below threshold. Thus the input data is converted into approval style outputs which are then summed over all choosers. This produces social choice rankings for all of the alternatives. The optimal threshold resolves the issue in approval voting of how to accurately divide the candidates into two groups. Since the OTSC system converts utilitarian style inputs to approval style outputs, OTSC is a utilitarian approval hybrid system. The hybridization resolves two issues: it makes the issue of interpersonal comparisons moot, and it gives each chooser an optimal strategy which, when undertaken by the system itself and not by the individual chooser, disincentivizes individual choosers from choosing insincerely. Although we assume no knowledge of polling statistics, the OTSC system is generalizable to the case in which polling information is known.

The issue of interpersonal comparisons is moot because any affine linear transformation of an individual's utility scale will produce the same results when processed by the OTSC system. If inputs are specified as preference rankings rather than ratings, the rankings can be converted to utility style ratings which can then be processed by the OTSC system. The outputs which are in the form of social rankings can also be converted back to ratings because the underlying utility information for each individual chooser is known. The utility of the social choice can be computed for each individual and for society as a whole.

Finally, we conjecture that the OTSC mechanism will produce the utilitarian winner(s), that is the winner(s) that maximize social utility since it has been shown by other writers such as Lehtinen (2015: p.35) that "strategic behavior increases the frequency with which the *utilitarian winner* is chosen compared to sincere behavior ". In the mechanism considered here, the strategy is implemented not by individual choosers but by the mechanism itself.

Arrow's main conclusion has been known since 1785 from the work of the Marquis de Condorcet, but Arrow attempted to elaborate and recast the paradox of voting as a proof that any kind of rational system which purports to determine the public good instead leads to a dictatorship which accorded nicely with Cold War philosophy directed at the Soviet Union. Alex Abella (2008: p. 49) wrote: “To combat the communist credo, postwar American intellectuals sought a version of history that eliminated once and for all the Marxist dogma: 'From each according to his ability, to each according to his needs.' The new doctrine would substitute the oppressive, omniscient Marxist state with a system that championed the right of individuals to make their own choices and their own mistakes. That doctrine, elaborated at RAND in 1950, was called rational choice; its main proponent, a twenty-nine-year-old economist named Kenneth Arrow."

The American and French revolutions of 1776 and 1789 respectively, although originally expressing their zeal for government by the people, ended up enshrining power in representative government precisely because the writers of their Constitutions did not trust the people. One of the most important theoreticians of the French revolution, the Abbe Sieyes, wrote (Harries-Jones, 2016: p. 78), “In a country that is not a democracy – and France cannot be one – the people, I repeat, can speak or act only through its representatives.” David Van Reybrouck writes (2016: pp. 89-91), “The French Revolution, like the American, did not dislodge the aristocracy to replace it with a democracy but rather dislodged a hereditary aristocracy to replace it with an elected aristocracy, *'une aristocratie elective'*, to use Rousseau's term.” The impossibility theorems of Arrow and Gibbard-Satterthwaite seem to have driven this point home since they claim that economic democracy and political direct democracy are impossible leaving only capitalist economics and representative democracy with a sound epistemic basis. In the American system of democracy, gerrymandering has insured that its representatives will indeed constitute *'une aristocratie elective'.* The work presented here proves that direct political and economic democracy do in fact have a sound scientific basis and that rational and normative social choice is indeed possible.

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